Walks, Trails and Circuits

Table of contents

Walks 1
Trails 2
Circuits 2
Connected 3
Common mistakes 4
See also 5

A walk is a way of getting around a graph: start at a vertex and keep moving along edges. A trail is a walk that never uses an edge twice, and a circuit is a trail that finishes where it started. The difference that matters is **edges versus vertices**: a trail may come back to a vertex it has already visited, as long as it leaves along a new edge. The last definition here, connected, says that a walk can get from any vertex to any other. These words are what euler-circuits-and-trails[1] and trees[2] are built from. Graphs, degree and the rest of the basic vocabulary are in graphs[3]. First met in 2026-10-26-walks-trails-and-circuits[4].

Walks

A **walk** is a sequence

$$v_0\, e_1\, v_1\, e_2\, v_2 \cdots v_{i-1}\, e_i\, v_i \cdots e_{n-1}\, v_{n-1}\, e_n\, v_n$$

where each v_i is a vertex, and each e_i is an edge with endpoints v_{i-1} and v_i.

It alternates vertex, edge, vertex, edge, …, and starts and ends with a vertex. Each edge joins the vertex before it to the vertex after it. This is a **walk from v_0 to v_n**, and it has n edges, counted with repeats.

[1] courses/math1061/euler-circuits-and-trails.pdf
[2] courses/math1061/trees.pdf
[3] courses/math1061/graphs.pdf
[4] courses/math1061/2026-10-26-walks-trails-and-circuits.pdf

- **Anything can repeat.** A walk can go back along the edge it just used, and pass through the same vertex many times.
- **A walk can have no edges.** With $n = 0$ the walk is the single vertex v_0. That is a walk from v_0 to itself.
- **A loop can be used.** If e is a loop at v, its endpoints are v and v, so $v\,e\,v$ is a walk.
- **Why the edges are written in.** If two vertices are joined by multiple edges, the vertices alone do not say which edge was used. In a simple graph two vertices are joined by at most one edge (graphs[5]), so the vertices determine the edges and a walk can be written $v_0 v_1 \cdots v_n$, or $a, b, c, \ldots$.

Trails

A **trail** is a walk where $e_1, e_2, \ldots, e_n$ are pairwise distinct.

No edge is used twice. That is the only condition. **Vertices may repeat**: a trail can come back to a vertex it has visited before, provided it arrives and leaves along edges it has not used yet. Every trail is a walk; a walk that uses some edge twice is not a trail.

Circuits

A **circuit** is a trail $v_0\,e_1\,v_1\,e_2 \cdots v_{n-1}\,e_n\,v_n$ where $v_0 = v_n$.

A circuit is a trail that finishes where it started. Like any trail it uses no edge twice, and it can pass through other vertices more than once.

A **trivial circuit** is a circuit consisting of a single vertex and no edges.
A **non-trivial circuit** is a circuit that has edges.
A graph is **circuit-free** if there are no non-trivial circuits.

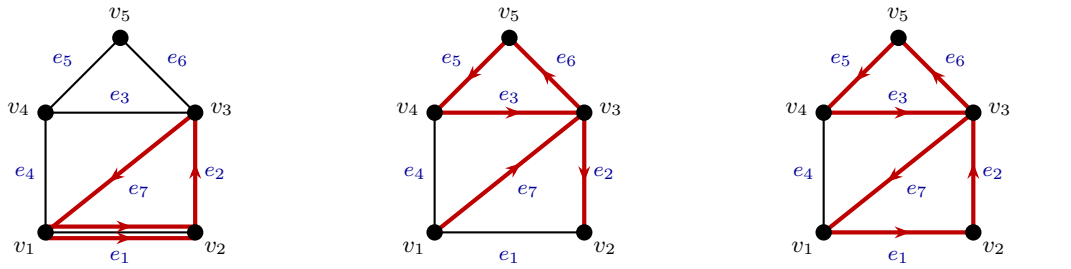
Every vertex v gives a trivial circuit, the walk v with $n = 0$: it is a trail (it has no edges to repeat) and it starts and ends at v. Every graph has these, so "circuit-free" has to mean free of the non-trivial ones.

The smallest non-trivial circuits:

- **One edge:** a loop e at v gives the circuit $v\,e\,v$. An ordinary edge e joining v and w does not give one, because $v\,e\,w\,e\,v$ uses e twice.
- **Two edges:** two edges e_1, e_2 that both join v and w give the circuit $v\,e_1\,w\,e_2\,v$.
- **In a simple graph** there are no loops and no multiple edges, so a non-trivial circuit has at least 3 edges. A triangle $a\,b\,c\,a$ is the smallest.

[5] courses/math1061/graphs.pdf

	Edge repeated?	Vertex repeated?	Ends where it starts?
Walk	allowed	allowed	allowed, not required
Trail	**no**	allowed	allowed, not required
Circuit	**no**	allowed	**yes**



walk, not a trail

$v_1\, e_1\, v_2\, e_2\, v_3\, e_7\, v_1\, e_1\, v_2$

e_1 used twice

trail, not a circuit

$v_1\, e_7\, v_3\, e_6\, v_5\, e_5\, v_4\, e_3\, v_3\, e_2\, v_2$

v_3 twice, no edge twice, ends at $v_2 \neq v_1$

circuit

$v_1\, e_1\, v_2\, e_2\, v_3\, e_6\, v_5\, e_5\, v_4\, e_3\, v_3\, e_7\, v_1$

v_3 twice, no edge twice, back at v_1

Figure 1: The three definitions on one graph, with the route in red and arrows showing the direction of travel. Left: a walk that is not a trail, since it uses e_1 twice, drawn as the doubled red line. Middle: a trail that is not a circuit. It visits v_3 twice, arriving along e_7 and e_3 and leaving along e_6 and e_2, but uses every edge at most once, and it ends at v_2, not at v_1 where it started. Right: a circuit. It also passes through v_3 twice, uses no edge twice, and returns to v_1.

Two things about circuits used later, in the proofs in euler-circuits-and-trails[6]:

- **A circuit can be started anywhere on it.** If $v_0\, e_1\, v_1 \cdots e_n\, v_n$ is a circuit and $1 \leq k \leq n$, then
$$v_k\, e_{k+1}\, v_{k+1} \cdots e_n\, v_n\, e_1\, v_1 \cdots e_k\, v_k$$
is a circuit too. Since $v_n = v_0$, each edge still joins the vertices beside it, it uses the same edges, and it starts and ends at v_k.
- **A circuit can be read backwards.** $v_n\, e_n\, v_{n-1} \cdots e_1\, v_0$ is a circuit with the same edges.

Connected

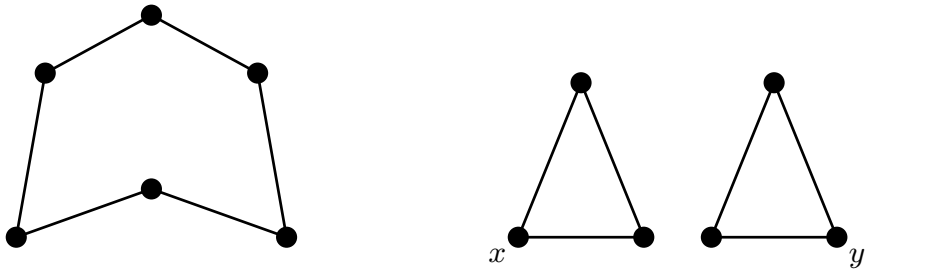
A graph is **connected** if and only if there is a walk from any given vertex to any other given vertex.

[6] courses/math1061/euler-circuits-and-trails.pdf

A graph that is not connected is **disconnected**: there are two vertices with no walk from one to the other. A drawing of a disconnected graph falls into separate pieces with no edge between them.

- **A graph with one vertex is connected.** There is no other vertex to reach.
- **An isolated vertex disconnects a graph** with two or more vertices. A walk from it would have to start along an edge, and it has none.
- **A crossing in a drawing is not a connection.** Two lines that cross without a dot do not meet, so a drawing that looks like one piece can be a disconnected graph.

To check, pick a vertex, mark every vertex that can be reached from it along edges, then everything reachable from those, and so on. The graph is connected exactly when every vertex ends up marked.



connected **disconnected**: no walk from x to y

Figure 2: Two graphs with the same degrees, six vertices of degree 2 each. Left: one circuit through all six vertices, a connected graph, since from any vertex there is a walk to any other around the circuit. Right: two separate triangles, a disconnected graph. There is no walk from x to y, since every walk from x stays inside its triangle. So the degrees alone do not decide whether a graph is connected.

Common mistakes

- **Thinking a trail cannot revisit a vertex.** A trail can pass through a vertex as often as it likes. It is the **edges** that may not repeat.
- **Thinking a circuit cannot revisit a vertex.** Same again: the circuit in the figure above passes through v_3 twice.
- **Going out and back along one edge and calling it a circuit.** $v\,e\,w\,e\,v$ ends where it starts, but uses e twice, so it is not a trail and not a circuit. It is only a walk.
- **Forgetting the trivial circuit.** A single vertex with no edges is a circuit. That is why circuit-free is defined by having no **non-trivial** circuits.
- **Reading a crossing as a vertex.** Only dots are vertices, so lines that cross do not join two pieces of a graph.

See also

- graphs[7] — graphs, loops, multiple edges, simple graphs and degree
- euler-circuits-and-trails[8] — a circuit or trail that uses every edge exactly once
- trees[9] — connected, circuit-free graphs
- 2026-10-26-walks-trails-and-circuits[10] — Lecture 34, where these definitions are introduced
- practice-problems[11] — §44, with full worked solutions

[7] courses/math1061/graphs.pdf
[8] courses/math1061/euler-circuits-and-trails.pdf
[9] courses/math1061/trees.pdf
[10] courses/math1061/2026-10-26-walks-trails-and-circuits.pdf
[11] courses/math1061/practice-problems.pdf