# Valid Argument Forms

## Table of contents



An **argument** is a sequence of statements: one or more **premises**, followed by a **conclusion**. An argument is **valid** if whenever all the premises are true, the conclusion is also true — equivalently, if

$$(\text{Premise}_1 \wedge \text{Premise}_2 \wedge \cdots \wedge \text{Premise}_n) \rightarrow \text{Conclusion}$$

is a tautology.

Validity is about the argument's *form*, not the truth of its statements: an argument can be valid with a false premise or a false conclusion. It can also have true premises and a true conclusion and still be invalid, if the truth of the conclusion doesn't actually follow from the premises.

> **Provided in the exam.** The named forms below are printed on the *MATH1061/MATH7861 Examination Formula Page*, alongside the logical-equivalence-laws[1] — so recognising which form an argument matches, and naming it, is the assessed skill rather than recalling the list.

**Why is an argument automatically valid if it has a false premise?** If any premise is false, the conjunction of the premises is false — and a false statement implies anything vacuously ($\mathbf{c} \rightarrow X$ is always T). So $(\text{Premise}_1 \wedge \cdots \wedge \text{Premise}_n) \rightarrow \text{Conclusion}$ is trivially a tautology regardless of the conclusion.

[1] courses/math1061/logical-equivalence-laws.pdf

### Determining validity

- **Truth table method**: build a truth table for the premises and the conclusion. The argument is valid iff every row where all premises are T also has the conclusion T.
- **Rules of inference method**: chain the laws of logical equivalence (see logical-equivalence-laws[2]) and the valid argument forms below to derive the conclusion from the premises.
- **Invalidity method**: assume all the premises are true and the conclusion is false, then reason out what that assumption forces each variable to be. Reaching a contradiction (some variable forced to be both T and F) means the assumption was impossible — the argument is **valid**. Reaching a consistent assignment instead gives a counterexample — the argument is **invalid**. (This directed search is usually faster than building the full truth table once there are 3+ variables.)

### Valid argument forms

| Name | Form(s) |
|---|---|
| Modus Ponens | $p \to q,\ p \therefore q$ |
| Modus Tollens | $p \to q,\ \sim q \therefore \sim p$ |
| Generalization | $p \therefore p \lor q$; or $q \therefore p \lor q$ |
| Specialization | $p \land q \therefore p$; or $p \land q \therefore q$ |
| Conjunction | $p,\ q \therefore p \land q$ |
| Elimination | $p \lor q,\ \sim q \therefore p$; or $p \lor q,\ \sim p \therefore q$ |
| Transitivity | $p \to q,\ q \to r \therefore p \to r$ |
| Proof by division into cases | $p \lor q,\ p \to r,\ q \to r \therefore r$ |
| Contradiction rule | $\sim p \to \mathbf{c} \therefore p$ |

These don't need to be memorised — they're a reference to check your reasoning against, not a checklist to recite.

### See also

- determining-argument-validity[3] — worked example applying both methods above
- logical-equivalence-laws[4]
- conditional-statements[5]
- practice-problems[6] — §4 Valid and Invalid Arguments, with full worked solutions

[2] courses/math1061/logical-equivalence-laws.pdf
[3] courses/math1061/determining-argument-validity.pdf
[4] courses/math1061/logical-equivalence-laws.pdf
[5] courses/math1061/conditional-statements.pdf
[6] courses/math1061/practice-problems.pdf