Logical Equivalence & Laws

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Two statement forms are **logically equivalent** ($\equiv$) if they have the same truth value for every combination of truth values of their variables — shown via a truth table, or by chaining the laws below. To show two forms are **not equivalent** ($\not\equiv$), exhibit one combination of truth values where they differ.

$\equiv$ *compares* statements (like $=$ compares numbers); it's not a connective itself.

Laws of logical equivalence

> **Provided in the exam.** These laws are printed on the *MATH1061/MATH7861 Examination Formula Page*, so they don't need memorising — the marks are in choosing and applying them, and in naming the one you used at each step. The same page also carries the valid-argument-forms[1] and the set identities.

Given statement variables p, q, r, a tautology $\mathbf{t}$, and a contradiction $\mathbf{c}$:

Law	Form 1	Form 2
Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double negative	$\sim (\sim p) \equiv p$	
Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
Universal bound	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$

[1] courses/math1061/valid-argument-forms.pdf

Law	Form 1	Form 2
De Morgan's	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations of $\mathbf{t}/\mathbf{c}$	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

De Morgan's Laws are especially important for negating "and"/"or" statements — e.g. the negation of "5 is divisible by 2 and 6 is divisible by 2" is "5 is not divisible by 2 or 6 is not divisible by 2".

See also

- logical-connectives[2]
- conditional-statements[3]
- practice-problems[4] — §2 Logical Equivalence, with full worked solutions

[2] courses/math1061/logical-connectives.pdf
[3] courses/math1061/conditional-statements.pdf
[4] courses/math1061/practice-problems.pdf