Determining Argument Validity — Worked Examples

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Worked examples applying the methods from valid-argument-forms[1] — truth table, rules of inference, and the invalidity/counterexample search — across every argument from Lecture 4, on both valid and invalid arguments.

The argument

Consider the argument:

> If today is Monday, then I am wearing a pink shirt. Today is Monday. Therefore, I am wearing a pink shirt.

Let p = "today is Monday" and q = "I am wearing a pink shirt". In symbolic form:

$$p \to q, \qquad p \qquad \therefore q$$

This is the Modus Ponens form (see valid-argument-forms[2]).

[1] courses/math1061/valid-argument-forms.pdf
[2] courses/math1061/valid-argument-forms.pdf

Method 1: checking rows of the truth table

Build a truth table for the premises ($p \to q$ and p):

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

An argument is valid iff **every row where all the premises are true also has the conclusion true**. The premises here are $p \to q$ and p — both true only in row 1 (p=T, q=T, $p \to q$=T). In that row the conclusion q is also T, and there's no other row where both premises hold. So the argument is **valid**.

(To show an argument is *invalid* by this method, you instead go looking for a row where every premise is T but the conclusion is F — that's the method used later in this lecture's Activity 2(b)/3.)

Method 2: checking the combined statement is a tautology

Equivalently, build a truth table for $((p \to q) \wedge p) \to q$ — the premises conjoined, implying the conclusion:

p	q	$p \to q$	$((p \to q) \wedge p) \to q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Every row is T, so the statement is a **tautology** — the argument is valid. This is the same conclusion as Method 1, just packaged as one compound formula instead of a row-by-row check across separate premise/conclusion columns.

A valid argument can still have a false conclusion

This argument is valid regardless of whether p and q are actually true today — validity only says "*if* the premises hold, the conclusion follows", not that the premises actually do hold. The premise "if today is Monday, then I am wearing a pink shirt" might well be false in reality, and the argument built from it is still valid: the form guarantees the conclusion given the premises, not the truth of the premises themselves.

Worked example 2: a three-variable argument

Consider the argument (Lecture 4, Activity 1):

> If wages are raised, buying increases. If there is a depression, buying does not increase. Therefore there is not a depression or wages are not raised.

Let w = "wages are raised", b = "buying increases", d = "there is a depression".

Translating to symbolic form

$$w \to b, \qquad d \to \sim b \qquad \therefore \sim d \vee \sim w$$

Truth table method

w	b	d	$w \to b$	$d \to \sim b$	$\sim d \vee \sim w$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Both premises ($w \to b$ and $d \to \sim b$) are true together in rows 2, 6, 7, 8 — and the conclusion $\sim d \vee \sim w$ is also T in every one of those rows. There's no row where both premises hold and the conclusion fails, so the argument is **valid**.

Rules of inference method

1. $w \to b$ (premise 1)
2. $d \to \sim b$ (premise 2)
3. $\sim (\sim b) \to \sim d$ (contrapositive of 2)
4. $b \to \sim d$ (double negative law, from 3)

5. $w \rightarrow \sim d$ (Transitivity, from 1 and 4 — see valid-argument-forms[3])
6. $\sim w \vee \sim d$ (definition of $\rightarrow$, from 5)
7. $\sim d \vee \sim w$ (commutativity, from 6) ■

Writing out the contrapositive step explicitly (line 3) before simplifying it (line 4) makes the justification for line 5's Transitivity step airtight — Transitivity needs the forms $w \rightarrow b$ and $b \rightarrow \sim d$ to line up exactly.

Invalidity method, as a third check

Suppose, for contradiction, that all the premises are true but the conclusion is false.

- Conclusion false $\Rightarrow \sim d \vee \sim w$ is false $\Rightarrow \sim d$ is false and $\sim w$ is false $\Rightarrow d$ is true and w is true.
- Premise 1 ($w \rightarrow b$) is true and w is true $\Rightarrow b$ is true.
- But now d is true and b is true (so $\sim b$ is false), which makes premise 2 ($d \rightarrow \sim b$) false — contradicting the assumption that all the premises are true.

No consistent assignment exists, so the argument is **valid** — agreeing with both methods above.

All three methods (truth table, rules of inference, invalidity search) agree: this argument is valid.

Worked example 3: an argument via Conjunction, distribution, and Modus Ponens

Consider the argument (Lecture 4, Activity 2 — Q1 from the Methods video pre-class questions):

$$p \rightarrow q, \qquad p \vee r, \qquad p \vee \sim r \qquad \therefore q$$

Rules of inference method

1. $p \rightarrow q$ (given premise)
2. $p \vee r$ (given premise)
3. $p \vee \sim r$ (given premise)
4. $(p \vee r) \wedge (p \vee \sim r)$ (from 2 and 3, Conjunction — see valid-argument-forms[4])
5. $p \vee (r \wedge \sim r)$ (from 4, distributive law — see logical-equivalence-laws[5])

[3] courses/math1061/valid-argument-forms.pdf
[4] courses/math1061/valid-argument-forms.pdf
[5] courses/math1061/logical-equivalence-laws.pdf

6. $p \vee \mathbf{c}$ (from 5, negation law: $r \wedge \sim r \equiv \mathbf{c}$)
7. p (from 6, identity law)
8. q (from 1 and 7, Modus Ponens) ■

The trick here is spotting that the two premises $p \vee r$ and $p \vee \sim r$ combine (via Conjunction, then distributing p out) into "p, or a contradiction" — which forces p to be true regardless of r.

Invalidity method, as a check

Suppose all the premises are true but the conclusion is false.

- Conclusion false $\Rightarrow$ q is false.
- Premise 1 ($p \to q$) is true and q is false $\Rightarrow$ p is false.
- Premise 2 ($p \vee r$) is true and p is false $\Rightarrow$ r is true.
- Premise 3 ($p \vee \sim r$) is true and p is false $\Rightarrow$ $\sim r$ is true, i.e. r is false.

r can't be both true and false — impossible. No such truth values exist, so the argument is **valid**, agreeing with the rules-of-inference proof above.

Worked example 4: an invalid argument

Not every argument is valid. Consider (Lecture 4, Activity 3 — Q2 from the Methods video pre-class questions):

$$p \to q, \qquad q \to r, \qquad \sim p \vee \sim q \qquad \therefore r$$

Invalidity method

Suppose all the premises are true but the conclusion is false.

- Conclusion false $\Rightarrow$ r is false.
- Premise 2 ($q \to r$) is true and r is false $\Rightarrow$ q is false.
- Premise 1 ($p \to q$) is true and q is false $\Rightarrow$ p is false.
- Premise 3 ($\sim p \vee \sim q$), checked against p false and q false: it's true. No contradiction so far.

This gives a **consistent** assignment — p, q, r all false — that makes every premise true and the conclusion false:

p	q	r	$p \to q$	$q \to r$	$\sim p \vee \sim q$	r
F	F	F	T	T	T	F

All three premises hold, the conclusion doesn't — that's a counterexample, so the argument is **invalid**.

Unlike the other arguments above, the search for a contradiction here *succeeds* in finding one instead — that's what distinguishes an invalid argument from a valid one under this method.

See also

- valid-argument-forms[6]
- logical-equivalence-laws[7]
- practice-problems[8] — §5 Methods for Determining Validity, with full worked solutions

[6] courses/math1061/valid-argument-forms.pdf
[7] courses/math1061/logical-equivalence-laws.pdf
[8] courses/math1061/practice-problems.pdf