

Conditional & Biconditional Statements

Table of contents

Conditional ($\rightarrow$)	1
Contrapositive, converse, inverse	1
Necessary and sufficient conditions	2
Biconditional ($\leftrightarrow$)	2
Order of operations	2
See also	3

Conditional ($\rightarrow$)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

By definition, $p \rightarrow q \equiv \sim p \vee q$ — this lets any law of logical equivalence (see logical-equivalence-laws¹) be applied to conditionals.

$p \rightarrow q$ can be phrased: “ p implies q ”, “if p then q ”, “ q if p ”, “ p is sufficient for q ”, “ p only if q ”, “ q is necessary for p ”.

Warning: $p \rightarrow q \not\equiv q \rightarrow p$.

Contrapositive, converse, inverse

¹courses/math1061/logical-equivalence-laws.pdf

Name	Form	Equivalent to
Conditional	$p \rightarrow q$	Contrapositive
Contrapositive	$\sim q \rightarrow \sim p$	Conditional
Converse	$q \rightarrow p$	Inverse
Inverse	$\sim p \rightarrow \sim q$	Converse

The conditional and its **contrapositive** are always logically equivalent. The **converse** and **inverse** are always logically equivalent to each other — but *not* to the original conditional.

Necessary and sufficient conditions

- p is a **necessary condition** for q “if $\sim p$ then $\sim q$ ” “if q then p ” “ q only if p ”.
- p is a **sufficient condition** for q “if p then q ” “ q if p ”.

Examples:

- Non-negative is **necessary but not sufficient** for being a perfect square (-9 isn’t a square; 7 is non-negative but not a square).
- Divisible by 4 is **sufficient but not necessary** for being even (8 : divisible by 4 even; 10 : even but not divisible by 4).

Biconditional ($\leftrightarrow$)

$$a \leftrightarrow b \equiv (a \rightarrow b) \wedge (b \rightarrow a)$$

“ a iff b ” “ a only if b , and a if b ” “ b is necessary and sufficient for a ”.

Order of operations

1. $\sim$
2. $\wedge, \vee$ (equal)
3. $\rightarrow, \leftrightarrow$ (equal)

Parenthesise explicitly rather than relying on left-to-right tie-breaking.

See also

- [logical-connectives](#)²
- [logical-equivalence-laws](#)³
- [practice-problems](#)⁴ — §3 Conditional Statements, with full worked solutions

²[courses/math1061/logical-connectives.pdf](#)

³[courses/math1061/logical-equivalence-laws.pdf](#)

⁴[courses/math1061/practice-problems.pdf](#)