Lecture 4 — Valid and Invalid Arguments

Table of contents

Pre-work: Video 004 (Valid & Invalid Arguments), Video 005 (Methods for Determining Validity). See valid-argument-forms[1] for the reference definitions/table this lecture builds on.

Learning goals

1. Understand the difference between a valid argument and an invalid argument.
2. Use a truth table to determine the validity of an argument.
3. Use rules of inference to demonstrate the validity of an argument.
4. Look for truth values that demonstrate an argument is invalid.

Valid vs invalid arguments

An argument is **valid** if the conjunction of its premises implies its conclusion — whenever all the premises are true, the conclusion is also true. See valid-argument-forms[2] for the formal definition and the standard forms (Modus Ponens, Modus Tollens, etc.), and determining-argument-validity[3] for a worked example walking through both the truth-table (row-checking) and tautology methods on the same argument.

Recall: $p \to q \equiv\sim p \vee q \equiv\sim q \to\sim p$.

[1] courses/math1061/valid-argument-forms.pdf
[2] courses/math1061/valid-argument-forms.pdf
[3] courses/math1061/determining-argument-validity.pdf

A valid argument can have a false conclusion — see determining-argument-validity[4] for why.

Do the rules of inference need to be memorised? No — valid-argument-forms[5] is a reference to check against, not something to recite.

In-class activities

Argument 1: Let $w =$ "wages are raised", $b =$ "buying increases", $d =$ "there is a depression".

> If wages are raised, buying increases. If there is a depression, buying does not increase. Therefore there is not a depression or wages are not raised.

- (a) Write this argument in symbolic form.
- (b) Use a truth table to determine whether the argument is valid.
- (c) Use the laws of logical equivalence and rules of inference to show the argument is valid.

Argument 2 (Q1 from the Methods video pre-class questions):

$$p \to q, \quad p \vee r, \quad p \vee \sim r \quad \therefore q$$

- (a) Use the laws of logical equivalence and rules of inference to show this argument is valid.
- (b) Alternatively, try to find truth values that make all the premises true but the conclusion false — if none exist, the argument is valid.

Argument 3 (Q2 from the Methods video pre-class questions):

$$p \to q, \quad q \to r, \quad \sim p \vee \sim q \quad \therefore r$$

Try to find truth values that make all the premises true but the conclusion false, to determine whether the argument is valid or invalid.

Extra practice: revisit Argument 1 — try to find truth values that make all the premises true but the conclusion false, to double check its validity via the invalidity method.

[4] courses/math1061/determining-argument-validity.pdf
[5] courses/math1061/valid-argument-forms.pdf

See also

- valid-argument-forms[6] — the named argument forms (modus ponens, modus tollens, elimination, generalisation, specialisation)
- determining-argument-validity[7] — the truth-table and invalidity methods worked through
- 2026-07-30-conditional-statements[8] — Lecture 3 · 2026-08-06-quantified-statements[9] — Lecture 5
- practice-problems[10] — §4 Valid and Invalid Arguments, with full worked solutions
- practice-problems[11] — §5 Methods for Determining Validity, with full worked solutions

[6] courses/math1061/valid-argument-forms.pdf
[7] courses/math1061/determining-argument-validity.pdf
[8] courses/math1061/2026-07-30-conditional-statements.pdf
[9] courses/math1061/2026-08-06-quantified-statements.pdf
[10] courses/math1061/practice-problems.pdf
[11] courses/math1061/practice-problems.pdf