Lecture 2 — Logical Form and Logical Equivalence

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Pre-work: Video 001 (Logical Form), Video 002 (Logical Equivalence). See logical-connectives[1] and logical-equivalence-laws[2] for the reference definitions/tables this lecture builds on.

Learning goals

1. Appreciate the difficulties in translating between English and mathematics.
2. Understand the logical connectives $\sim$ (not), $\wedge$ (and), $\vee$ (or).
3. Construct a truth table for a statement form in standard form.
4. Use a truth table to establish logical equivalence of two statement forms.
5. Apply logical equivalence laws to establish logical equivalence of two statement forms.

Statements

A **statement** (or **proposition**) is a sentence that is either true or false, but not both.

- Statements: "6 is odd.", "It is raining."

[1] courses/math1061/logical-connectives.pdf
[2] courses/math1061/logical-equivalence-laws.pdf

- Not statements: "x is odd." (depends on x), "Is it raining?" (a question), "She is happy." (not objectively T/F)

We use variables $p, q, r, ...$ to represent statements. See logical-connectives[3] for $\wedge, \vee, \sim$.

Truth tables

A truth table shows the truth value of a statement form for every combination of truth values of its variables. If a statement form has n variables, its truth table has 2^n rows.

Build truth tables in **standard form** (all combinations covered systematically, e.g. T/T/T, T/T/F, T/F/T, …) so your work is easy to check.

Inclusive vs. exclusive or

$\vee$ is **inclusive or**: $p \vee q$ is true when p is true, q is true, or both.

Exclusive or ($\oplus$, "one or the other but not both") is common in English ("Would you like red or white wine?") but is a different connective:

$$p \oplus q \equiv (p \vee q) \wedge (\sim (p \wedge q))$$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Order of operations

1. $\sim$ first.
2. $\wedge$ and $\vee$ equal second — use parentheses to disambiguate; otherwise left to right.

E.g. $(p \wedge q) \vee \sim (r \vee p) \wedge s$ means $((p \wedge q) \vee (\sim (r \vee p))) \wedge s$.

[3] courses/math1061/logical-connectives.pdf

Logical equivalence

Two statement forms are **logically equivalent** ($\equiv$) if they have the same truth value for every combination of inputs — shown via a truth table or the laws of logical equivalence (see logical-equivalence-laws[4]). To show they are **not** equivalent ($\not\equiv$), find one combination of truth values where they differ.

De Morgan's Laws (see logical-equivalence-laws[5] for the full table):

$$\sim (p \wedge q) \equiv \sim p \vee \sim q \qquad \sim (p \vee q) \equiv \sim p \wedge \sim q$$

In-class activities

Activity 1: Let p = "it is cold", q = "it is snowing", r = "it is sunny".

- Translate to symbolic form: (1) "It is not cold but it is snowing." (2) "It is neither snowing nor cold, but it is sunny."
- Translate to English: (1) $\sim q \wedge p$. (2) $(p \wedge q) \vee r$.
- Are "It is not cold but it is snowing" and "It is snowing and it is not cold" equivalent statements?

Activity 2: Construct a truth table for $(p \wedge \sim q) \vee (q \wedge r)$.

Activity 3: Show that $\sim (p \vee q) \not\equiv \sim p \vee \sim q$.

Activity 4: Write a statement logically equivalent to the negation of "5 is divisible by 2 and 6 is divisible by 2."

Activity 5: Use the laws of logical equivalence to prove:

- Q2: $(p \wedge \sim q) \wedge (\sim p \vee q) \equiv \mathbf{c}$ (a contradiction).
- Q3: $\sim p \vee (p \wedge q) \equiv \sim p \vee q$.

Activity 6 (carries into Lecture 3):

- (a) Use a truth table to show $\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \equiv p$.
- (b) Use the laws of logical equivalence to show the same.

[4] courses/math1061/logical-equivalence-laws.pdf
[5] courses/math1061/logical-equivalence-laws.pdf

See also

- logical-connectives[6] · logical-equivalence-laws[7] — the reference definitions and the full table of equivalence laws
- 2026-07-30-conditional-statements[8] — Lecture 3
- practice-problems[9] — §1 Logical Form, with full worked solutions
- practice-problems[10] — §2 Logical Equivalence, with full worked solutions

[6] courses/math1061/logical-connectives.pdf
[7] courses/math1061/logical-equivalence-laws.pdf
[8] courses/math1061/2026-07-30-conditional-statements.pdf
[9] courses/math1061/practice-problems.pdf
[10] courses/math1061/practice-problems.pdf